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COMPOSITE HEAT TRANSFER WITH THERMAL RADIATION IN NON-GREY MEDIUM PART I: INTERACTION OF RADIATION WITH CONDUCTION

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Abstract—The energy equation including thermal radiation is a non-linear high order integro-differential equation and the spectroscopic constants involved are usually complex functions of frequency. Accordingly, it is formidable to solve the equation rigorously. On this basis many investigators have introduced the assumption of the grey gas that spectroscopic constants are independent of wavelength. This assumption, however, might smear the essential feature of radiative heat transfer. Alternatively dividing a spectral band into parts of center and wings and estimating an appropriate effective absorption coefficient in each part, the opaque (Rosseland) approximation is applicable to the central part in a band and the transparent approximation to the part of wings. Such an analytical procedure reduces to simple treatment despite of taking into account of non-grey behaviour. The current study considers a simple interaction problem between conduction and radiation excluding the convection in the mediums. Numerical calculations are performed on carbon monoxide and carbon dioxide.

NOMENCLATURE

c, light velocity;
c_p, specific heat at constant pressure;
E_{bb}, blackbody emissive power;
E_n, exponential integral;
h, Planck's constant;
i, spectral intensity;
k, Boltzmann's constant;
P_1, equation (31);
P_2, equation (31);
q^R, radiative heat flux;
q^R_ν, spectral radiative heat flux;
R_ν, spectral radiosity;
s, geometrical length;
T, temperature;
T^*, temperature at $y = 0$;
T_0, temperature at $y = y_0$;
y, coordinate axis (Fig. 1);
y_0, distance between the two parallel walls.

Greek symbols
ε, emissivity;
ε_{lim}, limiting emissivity;
θ, non-dimensional temperature $(= T/T^*)$;
Θ, polar angle;
κ_ν, spectral absorption coefficient;
$\bar{\kappa}$ or κ, effective absorption coefficient;
λ, thermal conductivity;
λ_R, Rosseland mean free path;
μ, $= \cos\Theta$;
ν, frequency of light;
ν_a, ν_b, ν_c, ν_d, ν_0, $(\Delta\nu)_{op}$, ref. to Fig. 3;

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ρ, density;
σ, Stefan–Boltzmann constant;
τ, optical length;
ϕ, azimuthal angle;
$\boldsymbol{\Omega}$, solid angle.

Superscripts and subscripts
0, value of $y = y_0$;
ν, monochromatic;
bb, black body;
op., opaque;
tr., transparent;
*, standard value;
R, radiative;
$'$, differentiation with respect to parameter.

1. INTRODUCTION

THE THERMAL radiation is a sort of electromagnetic wave which is transferred by photons, and the wavelength range of thermal radiation is mostly located in an infrared, where such phenomena as absorption, re-emission and scattering are induced when radiation passes through a radiative medium.

When radiative heat transfer is compared with other modes of heat transfer, such as conductive and convective, the former is characterized by its fundamental nature as having an action-at-a-distance and selection rule for the frequency. Consequently, the energy equation which governs the temperature profile in the medium becomes a non-linear higher order integro-differential equation when the radiation interacts with conduction and/or convection. It is difficult to solve such an equation, especially when radiation interacts with convection, because the terms which are concerned with a flow field are involved in the energy equation which is to be solved simultaneously with the momentum equation. As the first step, the problem of the interaction of radiation and conduction is taken up for consideration here.

In order to solve the problem theoretically, some assumptions or approximations are usually introduced, and an assumption of grey gas medium is adopted most frequently as it leads to a mathematical brevity. The validity of grey gas solution is, however, still doubtful as the frequency characteristic which is peculiar to radiative heat transfer is neglected.

The work in this area, that is, the problems of heat transfer in a non-grey radiative medium between infinite parallel walls were performed analytically by Cess *et al.* [1] and Taylor [2]. The former analysis is performed under an assumption that the conductive heat transfer is negligible and the latter for the case of optically thin medium ($\tau_0 < 1$). Both authors predict that the results demonstrate the inadequacies of the grey gas model.

In this paper an analytical procedure dealing with the non-grey behaviour of the mediums is presented for the interaction problem between conduction and radiation.

2. THEORETICAL ANALYSIS

2.1 *Co-ordinate system and fundamental equation*

The physical model and co-ordinate system are illustrated in Fig. 1. It consists of the

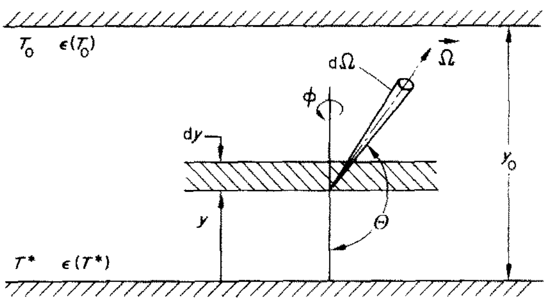

FIG. 1. Coordinate system.

infinite parallel walls of uniform temperature separated by a medium. The medium is assumed to be an absorbing, emitting, non-scattering and non-grey gas. The temperature of the wall at $y = 0$ is T^* and the temperature of the wall at $y = y_0$ is $T_0(T^* > T_0)$.

For the purpose of examining the effect of non-grey gas upon temperature field explicitly, the current study considers a simple interaction problem between conduction and radiation excluding the convection in the medium. The energy equation in the medium and the boundary conditions are

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y} - q_R\right) \quad (1)$$

$$\left.\begin{array}{ll} y = 0\,(\tau_\nu = 0): & T = T^* \\ y = y_0\,(\tau_\nu = \tau_{\nu 0}): & T = T_0 \end{array}\right\} \quad (2)$$

where

$$q^R = \int_0^\infty q_\nu^R(\tau_\nu)\,\mathrm{d}\nu. \quad (3)$$

Now, the spectral intensity of radiation i_ν in an absorbing-emitting and non-scattering medium is determined by Beer's law.

$$\frac{\mathrm{d}i_\nu}{\mathrm{d}s} = -\kappa_\nu i_\nu + \frac{\kappa_\nu E_{\mathrm{bb.}\nu}}{\pi}. \quad (4)$$

The first term and the second term of the right side of equation (4) indicate a variation of

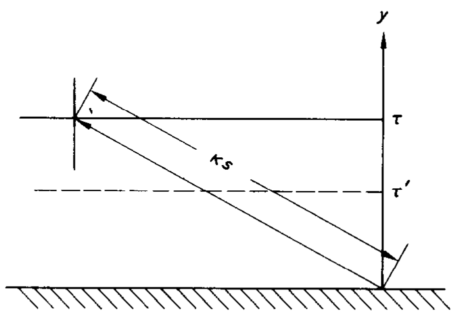

FIG. 2. Optical length.

radiative intensity due to absorption and emission respectively. Putting as indicated in Fig. 2.

$$\left.\begin{array}{l} \kappa_\nu \mathrm{d}s = \dfrac{1}{\mu}\mathrm{d}\tau \\ \mu = \cos\Theta \end{array}\right\} \quad (5)$$

then

$$\frac{\mathrm{d}i_\nu}{\mathrm{d}\tau} + \frac{i_\nu}{\mu} = \frac{1}{\mu}\frac{E_{\mathrm{bb.}\nu}}{\pi}. \quad (6)$$

The formal solution of equation (6) with the boundary conditions of equation (2) becomes

$$i_\nu(\tau) = i_\nu^* \exp\{-\tau/\mu\} + \int_0^\tau \frac{1}{\mu}\frac{E_{\mathrm{bb},\nu}}{\pi} \times \exp\{-(\tau-\tau')/\mu\}\,\mathrm{d}\tau' - i_{\nu 0}\exp\{-(\tau_0-\tau)/\mu\} - \int_\tau^{\tau_0} \frac{1}{\mu}\frac{E_{\mathrm{bb},\nu}}{\mu} \times \exp\{-(\tau'-\tau)/\mu\}\,\mathrm{d}\tau'. \quad (7)$$

The spectral heat flux of radiation q_ν^R is obtained by integration within a hemispherical solid angle.

$$q_\nu^R = \int_0^{2\pi}\int_0^{\pi/2} i_\nu \cos\Theta \sin\Theta\,\mathrm{d}\,\Theta\,\mathrm{d}\phi$$
$$= 2\Big[R_\nu(0)\,E_3(\tau_\nu) + \int_0^{\tau_\nu} E_{\mathrm{bb.}\nu}(\tau_\nu')\,E_2(\tau_\nu - \tau_\nu') \times \mathrm{d}\tau_\nu' - R_\nu(\tau_{\nu 0})\,E_3(\tau_{\nu 0} - \tau_\nu) - \int_{\tau_\nu}^{\tau_{\nu 0}} E_{\mathrm{bb.}\nu}(\tau_\nu')\,E_2(\tau_\nu' - \tau_\nu)\,\mathrm{d}\tau_\nu'\Big] \quad (8)$$

where $R_\nu(0)$, $R_\nu(\tau_{\nu 0})$, τ_ν and $E_n(\tau_\nu)$ are, in turn, wall radiosities at $y = 0$ and $y = \tau_{\nu 0}$, optical depth and exponential integral expressed in the following.

$$\left.\begin{array}{l} R_\nu(0) = \varepsilon_\nu(0)\,E_{\mathrm{bb.}\nu}(0) + 2[1-\varepsilon_\nu(0)] \\ \quad \times \Big[R_\nu(\tau_{\nu 0})E_3(\tau_{\nu 0}) + \displaystyle\int_0^{\tau_\nu} E_{\mathrm{bb.}\nu}(\tau_\nu')E_2(\tau_\nu')\mathrm{d}\tau_\nu'\Big] \\ R_\nu(\tau_{\nu_0}) = \varepsilon_\nu(\tau_{\nu_0})E_{\mathrm{bb.}\nu}(\tau_{\nu_0}) + 2[1-\varepsilon_\nu(\tau_{\nu_0})] \\ \quad \times \Big[R_\nu(0)E_3(\tau_{\nu_0}) + \displaystyle\int_0^{\tau_\nu} E_{\mathrm{bb.}\nu}(\tau_\nu')E_2(\tau_{\nu_0}-\tau_\nu')\mathrm{d}\tau_\nu'\Big] \end{array}\right\} \quad (9)$$

$$\tau_\nu = \int_0^y \kappa_\nu(y)\,\mathrm{d}y \quad (10)$$

$$E_n(\tau_\nu) = \int_0^1 \mu^{n-2}\exp(-\tau/\mu)\,\mathrm{d}\mu. \quad (11)$$

2.2 *The non-grey characteristics and the adoption of opaque and transparent approximations*

As already stated, the energy equation including thermal radiation is very complicated even in such a simple system as shown in Fig. 1 and, in addition, the spectroscopic constants are usually complex functions of frequency. Accordingly, it is formidable to solve the equation rigorously. On this basis many investigators have introduced the assumption of grey gas that spectroscopic constants are independent of frequency. The problem is, however, eventually cumbersome to resolve, and furthermore, there is the possibility of smearing the essential feature of radiative heat transfer.

When the optical thickness is extremely large or extremely small, the opaque approximation (Rosseland approximation) or the transparent approximation may be introduced. Alternatively whereas the spectra of real gases are banded and their band widths are generally narrow, the absorptivity or emissivity of gases, even with comparatively small optical length, is large and therefore in these frequency regions the spectral absorption coefficients or emission coefficients become substantially large. Thus, as schematically shown in Fig. 3, dividing the spectrum in the center and the part of wings and estimating an appropriate effective absorption coefficient in each part, the opaque approximation is applicable to the part of center and the transparent approximation to the part of wings. For the central part of the spectral band, the medium is optically so thick that the temperature of the medium varies very slowly for the length of $1/\kappa_\nu$ and, in consequence, $E_{bb.\nu}(\tau_\nu)$ is characteristic of the local temperature $T(\tau_\nu)$, as a first approximation, $E_{bb.\nu}(\tau'_\nu)$ may be expressed as

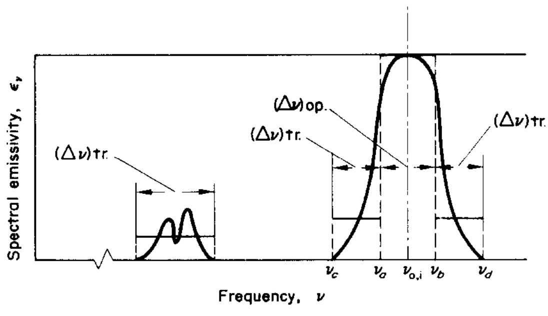

FIG. 3. Opaque and transparent approximation for vibration–rotation band.

$$E_{bb.\nu}(\tau'_\nu) = E_{bb.\nu}(\tau_\nu) + (\tau'_\nu - \tau_\nu)\, E'_{bb}(\tau_\nu) \qquad (12)$$

where the prime of $E'_{bb.\nu}$ denotes the differentiation with respect to τ'_ν. Substituting equation (12) into equation (8), the local monochromatic radiative heat flux is expressed as

$$\begin{aligned} q_\nu^R &= 2\Big[R_\nu(0)\,E_3(\tau_\nu) - R_\nu(\tau_{\nu0})\,E_3(\tau_{\nu0}-\tau_\nu) \\ &\quad + \int_0^{\tau_\nu} \{E_{bb.\nu}(\tau_\nu) + (\tau'_\nu - \tau_\nu)\,E'_{bb.\nu}(\tau_\nu)\} \\ &\quad \times E_2(\tau_\nu - \tau'_\nu)\,d\tau'_\nu - \int_{\tau_\nu}^{\tau_{\nu0}} \{E_{bb.\nu}(\tau_\nu) \\ &\quad + (\tau'_\nu - \tau_\nu)\,E'_{bb.\nu}(\tau'_\nu)\}\,E_2(\tau'_\nu - \tau_\nu)\,d\tau'_\nu\Big] \\ &= 2\Big[R_\nu(0)\,E_3(\tau_\nu) - R_\nu(\tau_{\nu0})\,E_3(\tau_{\nu0}-\tau_\nu) \\ &\quad + E_{bb.\nu}(\tau_\nu)\{-E_3(\tau_\nu) + E_3(\tau_{\nu0}-\tau_\nu)\} \\ &\quad + E'_{bb}(\tau_\nu)\{-\tfrac{2}{3} + \tau_\nu E_3(\tau_\nu) + E_4(\tau_\nu) \\ &\quad + (\tau_{\nu0}-\tau_\nu)E_3(\tau_{\nu0}-\tau_\nu) + E_4(\tau_{\nu0}-\tau_\nu)\}\Big] \qquad (13) \end{aligned}$$

for the optically thick medium, the following expressions are valid.

$$\tau, \tau_0 \to \infty \quad E_n(\tau) \to 0 \quad \tau^n E_n(\tau) \to 0. \qquad (14)$$

Then q_ν^R is simplified and the following equation is obtained.

$$q_\nu^R = -\frac{4}{3}\frac{dE_{bb.\nu}}{d\tau} = -\frac{4}{3}\frac{dE_{bb.\nu}}{dT}\frac{dT}{d\tau}. \qquad (15)$$

When the optical thickness is small, taking into account of the following equations,

$$\begin{aligned} E_2(\tau) &= 1 - 0(\tau) \\ E_3(\tau) &= 1 - \tau + 0(\tau^2) \end{aligned} \qquad (16)$$

one gets for the transparent approximation

$$q_\nu^R = 2[R_\nu(0)(\tfrac{1}{2} - \tau_\nu) + \int_0^{\tau_\nu} E_{bb.\nu}(\tau'_\nu)\,d\tau'_\nu - R_\nu(\tau_{\nu 0})\{\tfrac{1}{2} - (\tau_{\nu 0} - \tau)\} - \int_{\tau_\nu}^{\tau_{\nu 0}} E_{bb.\nu}(\tau'_\nu)\,d\tau'_\nu]. \quad (17)$$

Differentiating equation (17) with respect to τ_ν,

$$\frac{dq_\nu^R}{d\tau_\nu} = 2[-R_\nu(0) - R_\nu(\tau_{\nu_0}) + 2E_{bb.\nu}(\tau_\nu)]. \quad (18)$$

Generally the spectra of gases are banded whose widths are narrow and the emissivity (or absorptivity) within a band is large, in other words, the emission (absorption) coefficient is eventually large. Consequently as indicated in Fig. 3 dividing a band into central portion and wings, where the effective emission coefficients are introduced, it facilitates to solve the basic equation by applying the opaque and transparent approximations to the central part and wings within a band respectively. When there are two or more spectral bands, the radiative heat flux q^R is the sum of the individual radiative heat flux for each band.

$$q^R = \sum_i \left\{ \int_{(\Delta\nu)_{op\ i}} q_\nu^R\,d\nu + \int_{(\Delta\nu)_{tr,\ i}} q_\nu^R\,d\nu \right\}. \quad (19)$$

Substituting equation (19) into equation (1) one gets

$$\rho c_p \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial y^2} + \frac{4}{3}\sum_i \kappa_{op.i} \frac{\partial}{\partial \tau_{op.i}} \times \left(\int_{(\Delta\nu)_{op\ i}} \frac{dE_{bb,\nu}}{dT}\,d\nu \cdot \frac{dT}{d\tau_{op.i}} \right) + 2\sum_i \kappa_{tr.i} \int_{(\Delta\nu)_{tr\ i}} [R_\nu(0) + R_2(\tau_{tr.io}) - 2E_{bb,\nu}(\tau_{tr,i})]\,d\nu \quad (20)$$

where it is assumed that the absorption coefficients ($\kappa_{op.i}$ and $\kappa_{tr.i}$) are constant within the integral domains of $(\Delta\nu)_{op.i}$ and $(\Delta\nu)_{tr.i}$. Supposing that the absorption coefficient is constant over the entire range of $(\Delta\nu)_{op.i}$ and taking the case of $i = 1$ for simplicity, the radiative heat flux reduces to

$$q^R = -\frac{4}{3}\left[\int_{(\Delta\nu)_{op}} \frac{dE_{bb,\nu}}{dT}\,d\nu \right] \frac{dT}{d\tau}. \quad (21)$$

The above expression can be rewritten in an alternative form by introducing the limiting emissivity ε_{lim}.

$$q^R = -\frac{16}{3}\sigma T^3 \left(\varepsilon_{lim} + \frac{1}{4} T \frac{d\varepsilon_{lim}}{dT} \right) \frac{dT}{d\tau}. \quad (22)$$

The physical meaning of the limiting emissivity, as defined in the following equation,

$$\varepsilon_{lim} = \left(\int_{(\Delta\nu)_{op}} E_{bb,\nu}\,d\nu \right) \Big/ \left(\int_0^\infty E_{bb,\nu}\,d\nu \right) \quad (23)$$

is the radiative energy ratio of blackbody for frequency range $(\Delta\nu)_{op.}$ to σT^4, in other words, ε_{lim} means the band emissivity with infinite optical length. The substitution of equation (22) into (1) for steady state leads to

$$\frac{d}{dy}\left\{ \lambda \frac{dT}{dy} + \frac{16}{3}\frac{\sigma T^3}{\kappa}\left(\varepsilon_{lim} + \frac{1}{4} T \frac{d\varepsilon_{lim}}{dT} \right) \frac{dT}{d\tau} \right\} = 0 \quad (24)$$

or in the dimensionless form with boundary conditions

$$\frac{d}{d\tau}\left[\left\{ N + \frac{4}{3}\theta^3 \left(\varepsilon_{lim} + \frac{1}{4}\theta \frac{d\varepsilon_{lim}}{d\theta} \right) \right\} \frac{d\theta}{d\tau} \right] = 0 \quad (25)$$

$$\begin{aligned} \tau = 0 \quad &: \quad \theta = 1 \\ \tau = \tau_0 \quad &: \quad \theta = \theta_0 \end{aligned} \quad (26)$$

where the dimensionless variables and parameter are defined as follows.

$$\theta = T/T^*;\ \tau = \kappa y,\ N = \lambda\kappa/4\sigma T^{*3}. \quad (27)$$

Alternatively the black body emission $E_{bb,\nu}$ is expressed by Planck's equation or Wien's equation

$$E_{bb,2} = (2\pi h\nu^3/c^2)/\{\exp\frac{h\nu}{kT} - 1\}\text{(Planck)} \quad (28)$$

$$E_{bb,2} = (2\pi h\nu^3/c^2)/\exp\frac{h\nu}{kT} \qquad \text{(Wien)}. \quad (29)$$

The error involved in Wien's equation is less than one per cent in the range of $T/\nu \lesssim 10^{-1}\,s°K$ *or* $\lambda T \lesssim 0{\cdot}3\,cm°K$. For brevity substitution of equation (29) by Wien into equation (21) leads to

$$q^R = \frac{32\pi h}{3c^2T^*}\left[\nu_a^4 \exp\left(-\frac{1}{P_1\theta}\right)\right. \times \left\{\frac{1}{4\theta} + P_1 + 3P_1{}^2\theta + 6P_1{}^3\theta^2 + 6P_1{}^4\theta^3\right\} - \nu_b^4 \exp\left(-\frac{1}{P_2\theta}\right)\left\{\frac{1}{4\theta} + P_2 + 3P_2{}^2\theta \left. + 6P_2{}^3\theta^2 + 6P_2{}^4\theta^3\right\}\right]\frac{d\theta}{d\tau} \quad (30)$$

where

$$P_1 = kT^*/h\nu_a \quad P_2 = kT^*/h\nu_b. \quad (31)$$

Consequently the fundamental equation is expressed as

$$\frac{d}{d\tau}\left[\left[\left\{N + \frac{8\pi h}{3c^2\sigma T^{*4}}\left[\nu_a^4 \exp\left(-\frac{1}{P_1\theta}\right)\left\{\frac{1}{4\theta} + P_1 + 3P_1{}^2\theta + 6P_1{}^3\theta^2 + 6P_1{}^4\theta^3\right\} - \nu_b^4 \exp\left(-\frac{1}{P_2\theta}\right)\left\{\frac{1}{4\theta} + P_2 + 3P_2{}^2\theta + 6P_2{}^3\theta^2 + 6P_2{}^4\theta^3\right\}\right]\right\}\frac{d\theta}{d\tau}\right]\right] = 0. \quad (32)$$

Similarly in the range of frequency in which transparent approximation is applicable, radiative heat flux can be calculated. Unfortunately, it is difficult to calculate radiative heat fluxes by taking into consideration of the transparent approximation, because the rigorous spectroscopic data such as the frequency ranges of bands or the effective absorption coefficients for the radiative mediums of interest are ambiguous. Accordingly, for many engineering problems encountered practically it would be sufficient to correct the frequency width of central part of a band in a suitable way and simultaneously to couple with an appropriate estimation of the effective absorption coefficient, although this treatment is not applicable to the elaborate calculation.

For polyatomic molecules, there is the case when the radiative heat fluxes by many weak bands (overtones and combinations) are not always so small as to be neglected. In such a case, it is therefore necessary to add the heat fluxes by transparent approximations.

Alternatively, if the medium is grey, the limiting emissivity becomes constant and its value is unity and so the fundamental equation results in the familiar form

$$\frac{d}{d\tau}\left[\left(N + \frac{4}{3}\theta^3\right)\frac{d\theta}{d\tau}\right] = 0. \quad (33)$$

3. RESULTS OF CALCULATION

3.1 *For single vibration–rotation band with constant bandwidth*

As the second term of the fundamental equation is non-linear and a complicated function of temperature, it would be formidable to obtain an analytical solution of this problem. We therefore obtained a numerical solution by the method of Runge–Kutta–Gill with the aid of an electronic computer.

Carbon monoxide is adopted for the calculation, as it is one of the most simple radiative medium. That is, carbon monoxide is diatomic molecule and its frequency range of fundamental vibration ($\nu_a \sim \nu_b$) is from $6{\cdot}06 \times 10^{13}\,s^{-1}$ to $6{\cdot}69 \times 10^{13}\,s^{-1}$ (wave number range ($w_a \sim w_b$) is from $2020\,cm^{-1}$ to $2230\,cm^{-1}$), and the numerical value of the effective absorption coefficient κ, which varies proportionally accord- to the pressure of medium, is $2{\cdot}37\,cm^{-1}$ at 1 ata.

The calculation is performed by considering only the fundamental vibration, because the absorption coefficient of first overtone vibration is very small compared to that of the fundamental one.

Some typical calculation results are shown in Figs. 4–7. For a grey medium it is understood

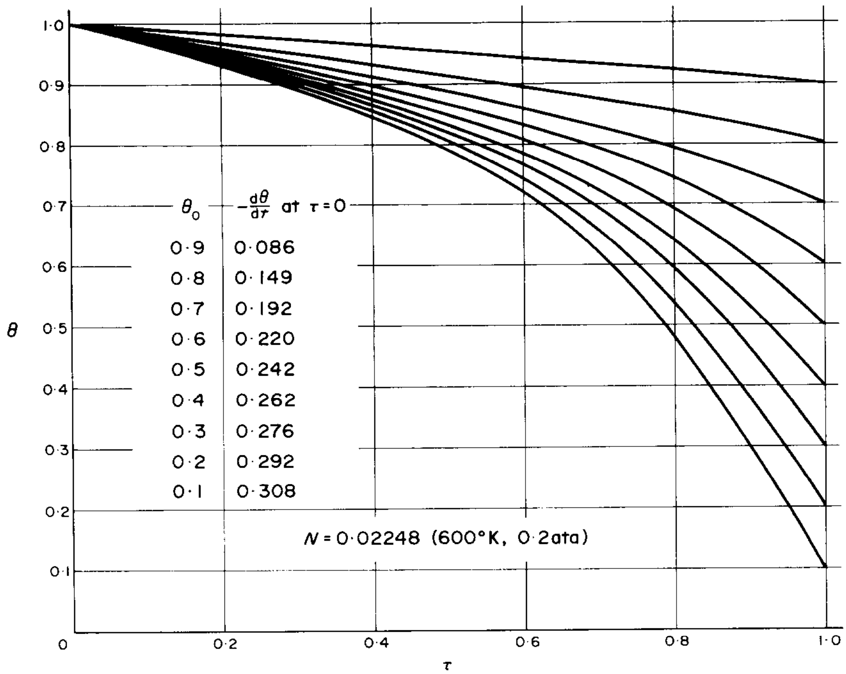

FIG. 4. Temperature profiles.

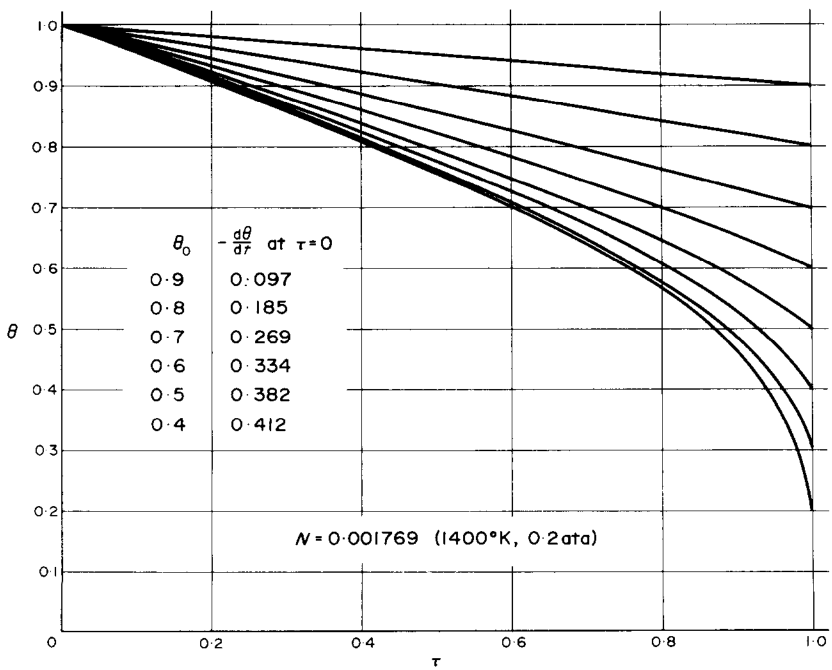

FIG. 5. Temperature profiles.

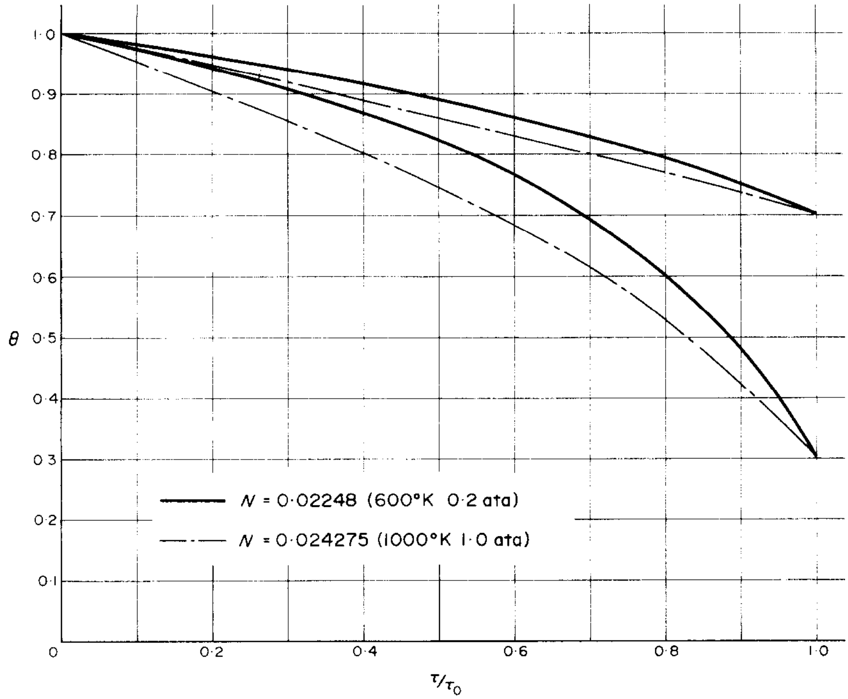

FIG. 6. Temperature profiles for constant N.

from equation (33) that the temperature profile is uniquely determined for a certain value of the interaction parameter N, however, for a non-grey medium the governing equation involves the limiting emissivity (ε_{lim}) and its derivative with respect to $T(\mathrm{d}\varepsilon_{\text{lim}}/\mathrm{d}T)$, as being shown in equation (25), so that the temperature profile depends not only on the interaction parameter N but the wall temperature T^*. Figure 6 shows the temperature distributions by varying the wall temperature T^* for constant N and reveals the fact that the temperature profile is more influenced at low wall temperature for constant N. In Figs. 4 and 5 the temperature profiles are drawn for constant pressure and at different wall temperature respectively (in this case the interaction parameter N differs). The comparison in these figures shows that the temperature profiles are not so much different except the temperature gradient for large temperature difference. Figure 7 illustrates also the temperature profiles for the different interaction parameter and the different boundary conditions at $\tau = \tau_0(\theta_0 = 0{\cdot}3$ and $\theta_0 = 0{\cdot}7)$ while T^* is held constant. Interaction parameter N denotes again the ratio of the conductive heat flux to the radiative one. As being expected the deviation of the temperature profile from the straight line passing through the points of $\theta^* = 1$ at $\tau = 0$ and of given θ_0 at $\tau = \tau_0$, which is a solution without radiation, tends to be large for small N and small θ_0. This means, in other words, that the influence of the radiation becomes large when the ratio of the conductive heat flux to the radiative is small and also when the temperature difference of the two walls is large.

Figure 8 represents the exact solution [3] by the grey gas approximation and simultaneously the solution including the characteristics of non-grey gas behaviour is reproduced for

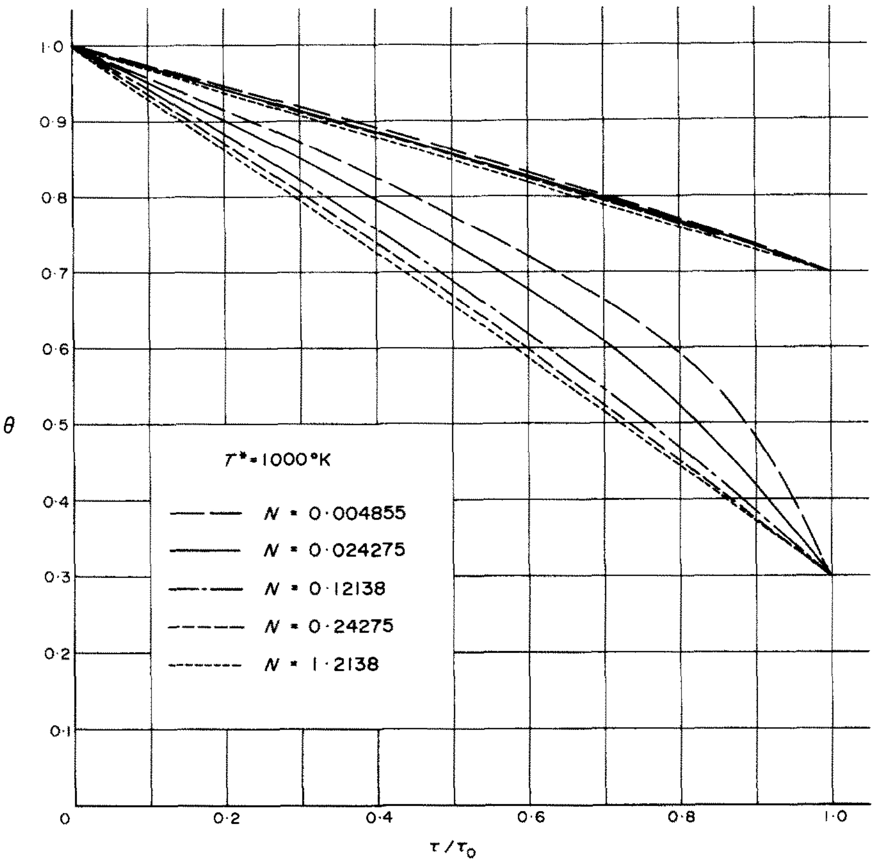

FIG. 7. Effect of N on temperature profiles.

comparison and further involves the solution of equation (33). The absorption coefficient of grey gas designates so-called Planck mean absorption coefficient expressed by the following equation

$$\bar{\kappa}_p = \int_0^\infty \kappa(\nu) E_{bb.\nu}\, d\nu \Big/ \int_0^\infty E_{bb.\nu}\, d\nu. \qquad (34)$$

The absorption coefficient of non-grey gas is the effective absorption coefficient defined by a tractable procedure within the frequency range of a band $\Delta\nu$. Assuming that $\kappa(\nu)$ is a stepwise function of frequency i.e. constant in the frequency range of band and zero out of the range, $\bar{\kappa}_p$ is expressed as follows.

$$\bar{\kappa}_p = \kappa \int_{\Delta\nu} E_{bb.\nu}\, d\nu \Big/ \int_0^\infty E_{bb.\nu}\, d\nu = \kappa\varepsilon_{lim}. \qquad (35)$$

Therefore, the translation of the interaction parameter N of the non-grey gas to the corresponding interaction parameter N of the grey gas is performed by substituting Planck absorption coefficient $\bar{\kappa}_p$ instead of the effective absorption coefficient κ. Although the limiting emissivity ε_{lim} varies considerably with temperature and, in consequence, Planckian absorption coefficient also varies with temperature, it is assumed that limiting emissivity is constant for simplicity and its numerical value is $6{\cdot}35 \times 10^{-2}$. In Fig. 8 the interaction parameter N for non-grey is translated by the

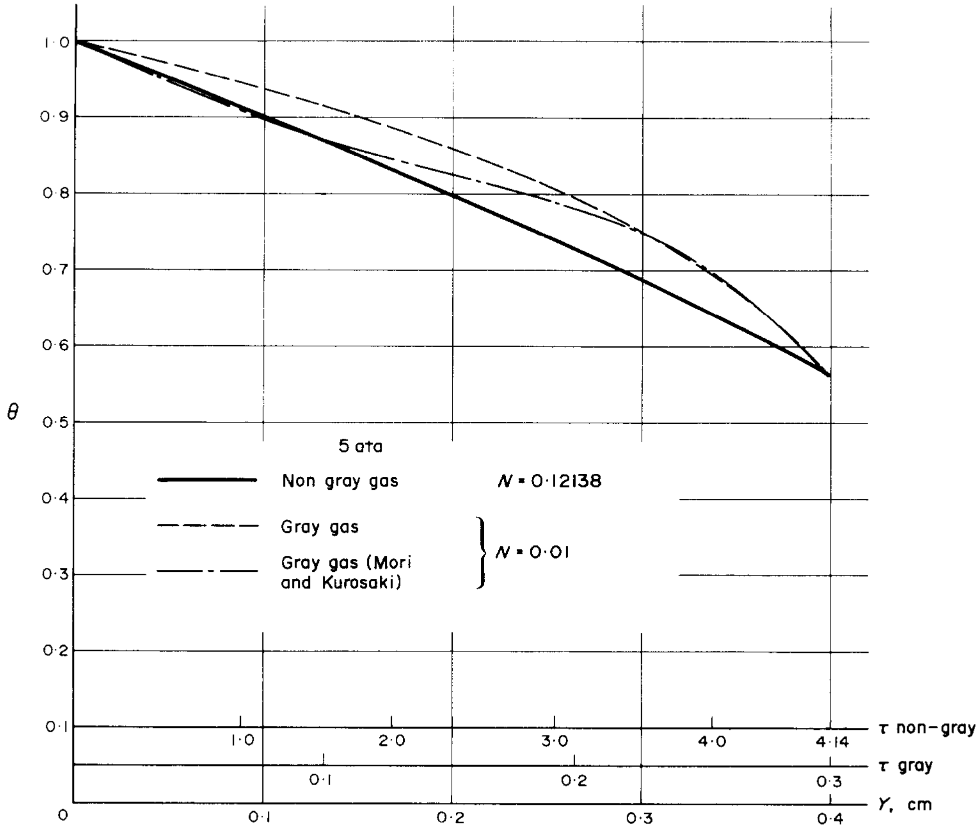

FIG. 8. Comparison of results for grey-gas and non-grey gas.

above procedure. In this case the optical lengths of grey and non-grey are different for the same geometric length, that is, the value of optical length in non-grey medium is much larger than that in grey for the same geometry and therefore this treatment of non-grey is substantially applicable to the magnitude of boundary layer thickness. The temperature profile of non-grey gas deviates less than that of grey gas from the straight line (pure conduction solution) and, consequently, the interaction of radiation and conduction of non-grey gas seems not to be so strong as that of grey gas. It might be explained by the fact that for a non-grey medium the radiative heat flux is confined in a narrow range of wavelength and then the interaction of radiation to conduction is rather weak. Qualitatively speaking the comparison of absorptivities between the grey and non-grey bodies by the corresponding absorption coefficients expressed in equation (35) shows that the absorptivity of non-grey body is smaller than that of grey. Denoting the absorptivities of non-grey and grey bodies by α, α_{grey} and the geometric length by s, α and α_{grey} are expressed by

$$\alpha = \{1 - \exp(-\kappa s)\}\,\varepsilon_{\lim} \tag{36}$$

$$\alpha_{grey} = 1 - \exp(-\kappa\varepsilon_{\lim}\,s). \tag{37}$$

As $\varepsilon_{\lim}$ is much less than unity, α is generally smaller than α grey in the range of practical interest. Another pronounced feature is that the temperature profile with Rosseland approximation do not vary with τ_0, though the exact solution of grey approximation is changed by τ_0.

3.2 *For a single vibration–rotation band with varying bandwidth*

The configuration and bandwidth of the spectra of real gases are changed in a complicated fashion by the variation of temperature. As a general trend the bandwidth of spectral band spreads as the temperature increases. Herein, the variation of temperature profile due to the bandwidth will be examined for carbon monoxide molecule. The bandwidth ($\Delta\nu = \nu_2 - \nu_1$) is expressed in the following,

$$\left.\begin{aligned} \nu_1 &= \nu_0 - 0{\cdot}000315T \times 10^{13}\,(\mathrm{s}^{-1}) \\ \nu_2 &= \nu_0 + 0{\cdot}000315T \times 10^{13}\,(\mathrm{s}^{-1}) \\ \nu_0 &= 6{\cdot}375 \times 10^{13} \end{aligned}\right\} \quad (38)$$

provided that the bandwidth is proportional to the absolute temperature, as indicated in Fig. 9. Figure 10 shows the comparison of the result of the variable bandwidth to that of constant bandwidth. Some examinations on these results reveal that the deviation of the temperature profile from that of constant bandwidth is merely due to the change of radiant energy by the broadening of bandwidth. The difference in temperature profiles is indistinguishable when the interaction parameter N is small and T^* is high.

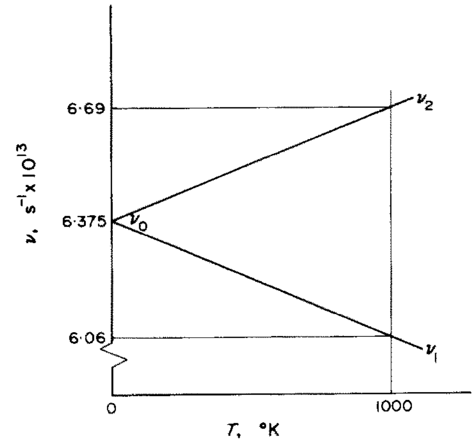

FIG. 9. Bandwidth variation.

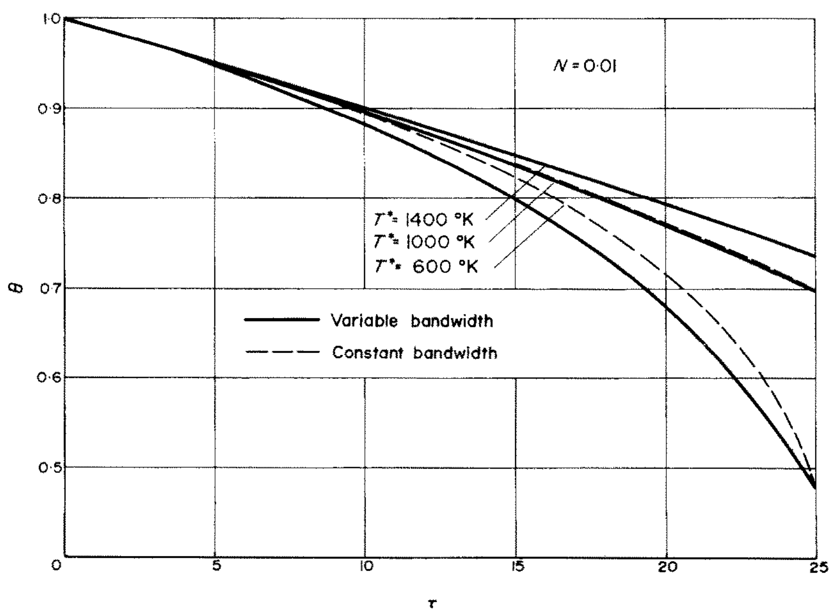

FIG. 10. Temperature profiles for variable bandwidth and constant bandwidth.

3.3 *For two vibration–rotation bands*

For tri-atomic molecules such as carbon dioxide and water vapor or for other poly-atomic molecules, one has to take into consideration of a number of vib–rot bands. Even for di-atomic molecule there exists pronounced effect by overtone band at a certain range of high temperature. In this section a tentative model for analysis is introduced, assuming that there are two bands around $4{\cdot}3\mu (6{\cdot}25 \times 10^{13} \sim 7{\cdot}89 \times 10^{3}\,s^{-1})$ and $15\mu (1{\cdot}62 \times 10^{13} \sim 2{\cdot}61 \times 10^{13}\,s^{-1})$ of wavelength (associating CO_2 molecule) and that the effective absorption coefficient of each band is same. The basic equation by taking account of the model mentioned above yields

$$\frac{d}{d\tau}\left[\left\{N + \frac{4}{3}\theta^3\left(\varepsilon_{\lim_{4,3\mu}} + \varepsilon_{\lim_{15\mu}} + \frac{1}{4}\theta\frac{d(\varepsilon_{\lim_{4,3\mu}} + \varepsilon_{\lim_{15\mu}})}{d\theta}\right)\right\}\right] = 0 \quad (39)$$

where $\varepsilon_{\lim_{4\cdot3\mu}}$ and $\varepsilon_{\lim_{15\mu}}$ are limiting emissivities of $4{\cdot}3\mu$ and 15μ bands as shown in Fig. 11. The temperature profile and gradient are illustrated in Figs. 12–14. The solid lines denote the results of two vib–rot bands, while the broken and chain lines are related to a single band for $4{\cdot}3\mu$ and 15μ band respectively. The result of a single band of $4{\cdot}3\mu$ is quite similar to that of CO molecule because of the location of band, however, the trend is a little different for 15μ band. It is worth having a closer look at the variation of the temperature gradient. For a 15μ band there exists a criterion where the temperature gradient decreases along τ. In other words, the radiant behavior of non-grey gas is attributed to the location of a band. For a gas having two bands as the limiting emissivity being additive, the resulant limiting emissivity becomes rather smoother curve with respect to temperature. In consequence, the derivative of $\varepsilon_{\lim}$ with temperature $(d\varepsilon_{\lim}/d\theta)$ results in minor effect so that one can expect that the temperature profile becomes to be resembled to that of grey medium. In fact, the calculation results demonstrates that the temperature gradient of a gas with two vib–rot bands is monotonously increasing as τ increases.

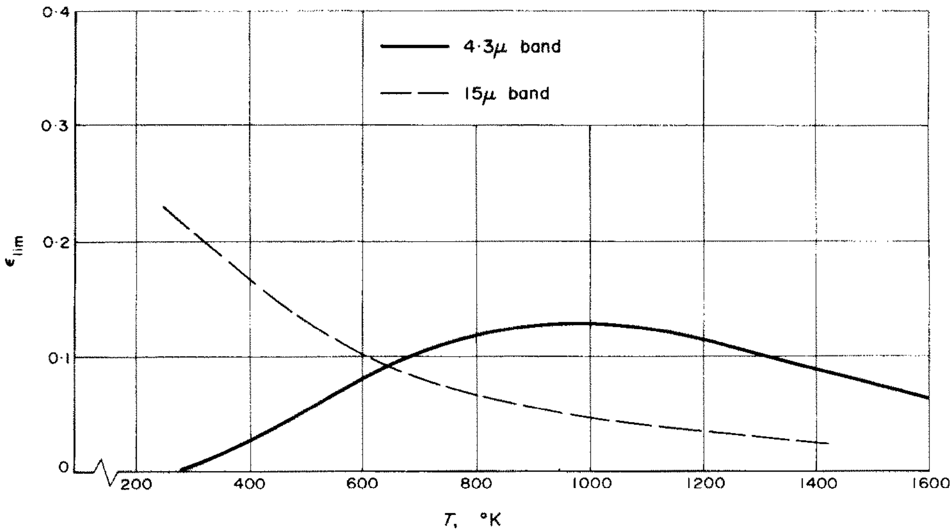

FIG. 11. Limiting emissivity.

4. SUMMARY

In current study an analytical technique con-

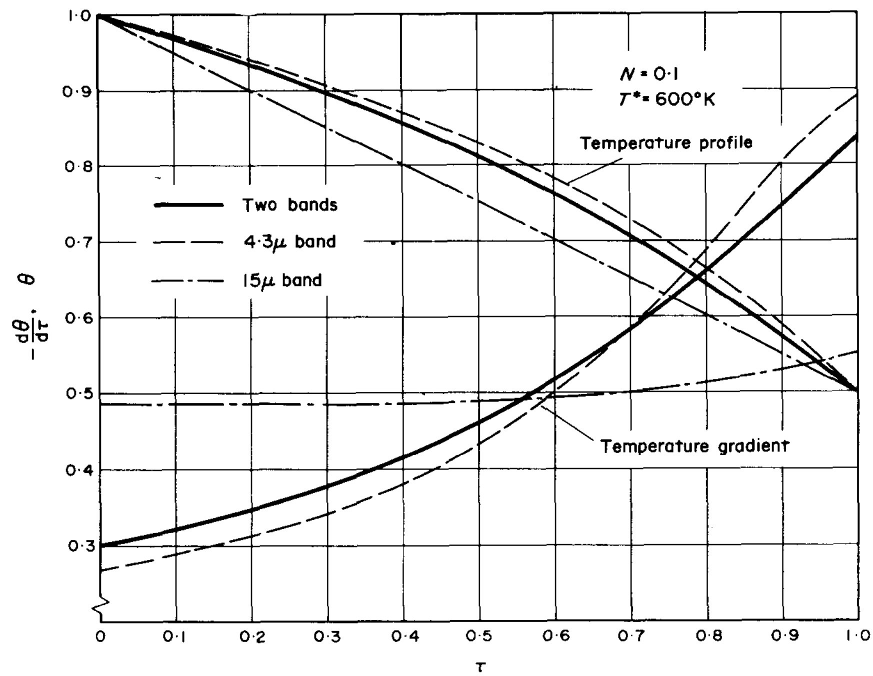

FIG. 12. Comparison of results for two bands and a single band.

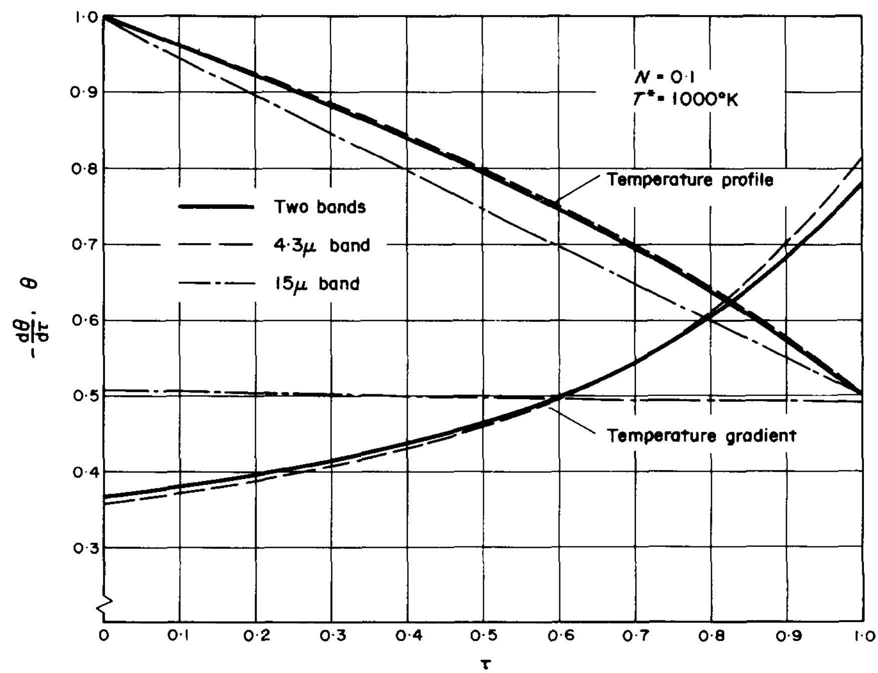

FIG. 13. Comparison of results for two bands and a single band.

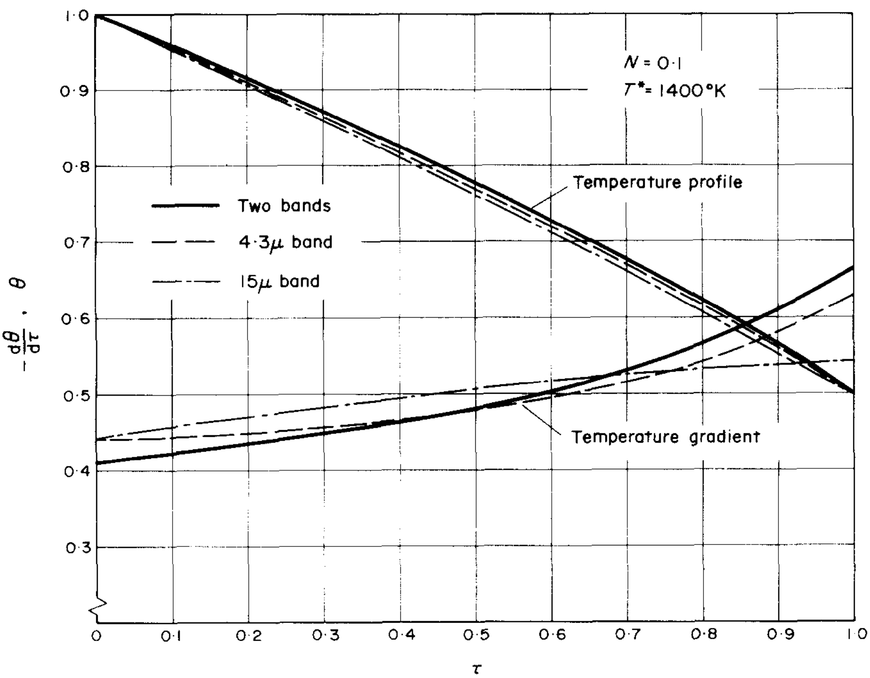

FIG. 14. Comparison of results for two bands and a single band.

cerning the treatment on the non-grey medium in the composite heat transfer of conduction and radiation is developed. In consequence, it facilitates the analytical prodecure by applying the opaque approximation within a band, despite of considering non-grey behavior which is the fundamental nature in radiative heat transfer. Numerical calculations are performed for simple molecules such as carbon monoxide and carbon dioxide as the illustrations of single and two vibration–rotation band model. As concluding remarks the location of band is of importance in radiative heat transfer and the broadening of band with temperature does not influence on temperature profile and finally the interaction of the radiation with conduction is not appreciable as in grey gas.

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TRANSFERT THERMIQUE COMPOSÉ AVEC RAYONNEMENT DANS UN MILIEU NON GRIS. 1ÈRE PARTIE: INTERACTION DU RAYONNEMENT ET DE LA CONDUCTION

Résumé—L'équation d'énergie incluant le rayonnement thermique est une équation intégro-différentielle non linéaire d'ordre élevé et les constantes spectroscopiques impliquées sont des fonctions compliquées de la fréquence. Par suite, il est malaisé de résoudre rigoureusement cette équation. Aussi de nombreux chercheurs ont ils introduit l'hypothèse d'un gaz gris dont les constantes spectroscopiques sont indépendantes de la longueur d'onde. Cette hypothèse peut brouiller la contribution essentielle du rayonnement thermique. En divisant une bande spectrale en régions du centre et des ailes et en estimant dans chacune d'elles un coefficient d'absorption effective approprié, une approximation d'opacité est applicable à une bande de la partie centrale et l'approximation de transparence à une partie des ailes. Ainsi une procédure analytique réduit à un traitement simple en dépit de la prise en compte d'un comportement non gris. L'étude considère un problème d'interaction simple entre conduction et rayonnement excluant la convection dans le milieu. Des calculs numériques sont conduits sur l'oxyde de carbone et le gaz carbonique.

ZUSAMMENGESETZTER WÄRMETRANSPORT MIT THERMISCHER STRAHLUNG IN EINEM NICHT GRAUEN MEDIUM. (TEIL I: WECHSELWIRKUNG VON STRAHLUNG MIT LEITUNG)

Zusammenfassung—Die Energiegleichung, die die thermische Strahlung enthält, ist eine nichtlineare Integrodifferentialgleichung höherer Ordnung, und die spektroskopischen Konstanten, die darin auftreten, sind normalerweise komplexe Funktionen der Frequenz. Daher ist es mühsam, die Gleichung exakt zu lösen. Aus diesem Grund haben viele Forscher die Annahme des grauen Gases eingeführt, dessen spektroskopische Konstanten von der Wellenlänge unabhängig sind. Diese Annahme könnte jedoch die wesentlichen Momente des Wärmetransportes durch Strahlung verwischen. Falls man aber ein Spektralband in Zentral- und Flügelteile zerlegt und passende effektive Absorptionskoeffizienten für jeden Teil schätzt, kann die undurchsichtige Näherung (Roseland) auf den Zentralteil eines Bandes und die transparente Näherung auf die Flügelteile angewandt werden. Dieses analytische Vorgehen führt zu einer einfachen Behandlung und trägt trotzdem dem nichtgrauen Verhalten Rechnung. Diese Untersuchung betrachtet ein einfaches Wechselwirkungsproblem zwischen Leitung und Strahlung, wobei die Konvektion in dem Medium ausgeschlossen ist. Numerische Rechnungen werden für Kohlenmonoxid und Kohlendioxid ausgeführt.

СЛОЖНЫЙ ТЕПЛООБМЕН ПРИ НАЛИЧИИ ТЕПЛОВОГО ИЗЛУЧЕНИЯ В НЕСЕРОЙ СРЕДЕ

Аннотация—Уравнение энергии для теплового излучения является нелинейным интегро-дифференциальным уравнением высокого порядка, а используемые спектроскопические постоянные обычно являются сложными функциями частоты. Естественно, довольно трудно точно решить такое уравнение. В следствии этого, многие исследователи вводили допущение о сером газе, которое заключалось в том, что спектроскопические постоянные не зависят от длины волны. Однако, это допущение могло бы сгладить существенную черту лучистого теплообмена. Разбивая спектральную полосу на центральную и боковые части и определяя соответствующий эффективный коэффициент поглощения в каждой части, можно применить приближение Росселанда непрозрачной среды для центральной части полосы и приближение прозрачной среды для боковых частей. Такой аналитический подход сводится к простой математической формулировке, несмотря на учёт поведения несерой среды. Настоящее исследование рассматривает задачу для простого случая взаимодействия между проводимостью, излучением и конвекцией в средах. Численные расчёты выполнены на окиси и двуокиси углерода.